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UNDERWATER EXPLOSION PHENOMENA: THE PARAMETERS OF MIGRATING BUBBLES

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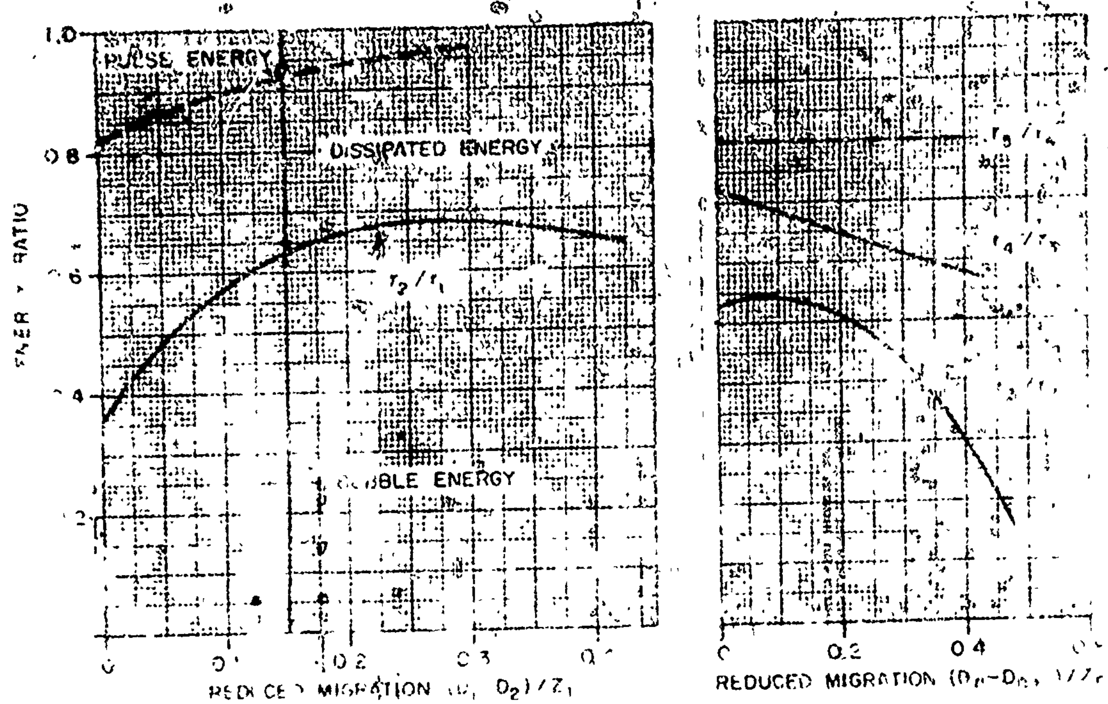

FIG. 6. CALCULATED BUBBLE ENERGY RATIOS.

The abscissae are a measure of the intensity of migration between the nth and (n+1)th cycle. The subscripts 1, 2, . n, ..5 refer to the cycle of pulsation. The dashed curve in the left graph is the ratio bubble pulse-energy to bubble energy plotted downward from the unity ordinate. The two curves divide this graph into three bands which illustrate the partition into the three dimensionless energy terms indicated.

UNDERWATER EXPLOSION PHENOMENA: THE PARAMETERS OF MIGRATING BUBBLES

by
Hans G. Sn[illegible]

ABSTRACT: The migration of underwater explosion bubbles caused by [illegible] y (buoyancy) affects the energy of the pulsation as well as period an[illegible] mum radius in the second and subsequent cycles. Experimental data on [illegible] of explosions in various depths are analyzed, and the bubble energy is [illegible] ined for five cycles of the oscillation as a function of the strength of [illegible] tion. The result is given in dimensionless form which permits the calcula[illegible] f periods, maximum radii, and migration for a wide range of condition[illegible]

An energy balance shows the surprising result that the bubble energ[illegible] he second cycle increases with increasing intensity of migration until [illegible] imum is reached at a condition of strong migration. Beyond this point, [illegible] ergy decreases again. This gain is found to be due to the decrease of t[illegible] rgy radiated by the bubble pulse.

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EXPLOSIONS RESEARCH DEPARTMENT
U. S. NAVAL ORDNANCE LABORATORY
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Preliminary results of this study have been reported in previous Although this final version shows little differences, the previou the bubble energy ratios must be considered to be superseded.

This report is part of a comprehensive study on the behavior of e bubbles. The work has been carried out under Task No. 301-664/43 and RUME-3-E-000/212-1/WFC08-10-004 PA 002.

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NAVORD REPORT 4185 12 October 1962

UNDERWATER EXPLOSION PHENOMENA: THE PARAMETERS OF MIGRATING BUBBLES

In this report the attempt is made to further the understanding of the various hydrodynamic processes associated with migrating explosion bubbles. Revealing results are obtained from an evaluation of experimental observations without the use of involved theoretical calculations. The important outcome of the calculations is a graph showing the bubble energy ratios for the first five cycles of the oscillation. Combined with simple equations this information permits the calculation of the bubble parameters for almost all practically important conditions of common explosives detonated under water. The forthcoming NOLTR 62-184 utilizes the results of this paper and presents a simple method for expedient calculations.

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The author wishes to express his appreciation for the helpful assistance of Miss R. V. Tipton in the computational efforts of this project.

I. INTRODUCTION

There is an extensive literature on the pulsation of bubbles produced by underwater explosions, listed in references (a) and (b)*. Almost all of this work refers to non-migrating bubbles, i.e. it is assumed that the center of the bubble remains stationary. This assumption is valid for small charges exploded in great depth, e.g. 1 lb in 500 ft. The bubbles of larger explosions move upward because of their buoyancy and thereby change the characteristics of the pulsation parameters. Little is known today about the fundamental behavior of such bubbles in the second and the following cycles.

A theoretical treatment of this problem is difficult for two reasons: (a) Migrating bubbles change their shape. When contracting, the lower interface of the originally spherical bubble moves faster inward than the upper interface. At an intermediate moment, the cross section of the bubble resembles that of a kidney. Later, the upper and lower interfaces collide and the bubble becomes a torus. Upon re-expansion, the spherical shape is roughly restored, but energy has been dissipated by the impact of the interfaces. (b) Near the bubble minimum, the gas-water interface becomes unstable. It tends to dissolve into a water spray which is projected into the interior of the bubble. This brings forth a cooling of the explosion gases and, thus, again a dissipation of energy.

Both of these dissipative processes are difficult to account for theoretically. A further portion of the bubble energy is radiated by the bubble pulse. These energies are not available for the subsequent cycles of the bubble pulsation. A knowledge of the remaining energy is of prime importance in any quantitative calculation of the later bubble phenomena.

A rather crude approach is used in this paper to find the bubble energy in each cycle of the oscillation. The analysis is based on the change of the bubble period caused by the various degrees of migration. Strong migration carries the bubble into a shallower depth. As a consequence the period of the next oscillation is increased when compared with that of a non-migrating bubble. To carry out the analysis, the magnitude of the bubble migration must be known. This process has been experimentally studied by means of acoustic ranging of

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* The list of references is found at the end of this report.

the bubble minimum. Further evidence is obtained in this paper from the observation of the time at which the bubble breaks the water surface. Once a bubble migrates into the proximity of the water surface, the emission of the bubble pulse ceases. The evidence whether or not a bubble pulse is observed before the breakthrough yields information on the depth of the preceding bubble maximum.

II. EQUATION FOR THE BUBBLE ENERGY RATIO

The period of the bubble oscillation in the n-th cycle is given by

$$(1) \qquad T_n = 0.374 \ t \frac{(r_n QW)^{1/3}}{Z_n^{5/6}} \left(1 - \alpha \frac{A_{Mn}}{D_n}\right) .$$

The symbols are explained in the list at the end of this report. The first portion of (1) is given in reference (b), where rQ is denoted by ϵ. The last term in the parenthesis accounts for the effect of the water surface which shortens the period This term will be discussed in paragraph III. The effect of the bottom of the sea is not included in (1), because the tests which we will analyze were made in water sufficiently deep as to make this effect negligibly small.

Forming the ratio of the n-th to the first period, we have

$$(2) \qquad \frac{T_n}{T_1} = \left(\frac{r_n}{r_1}\right)^{1/3} \left(\frac{Z_1}{Z_n}\right)^{5/6} \frac{1 - \alpha A_{Mn}/D_n}{1 - \alpha A_{M1}/D_1} .$$

In this equation, the dimensionless period t has been cancelled which amounts to the tacit assumption that it is the same for all cycles of the pulsation. On the basis of the classic bubble theory, this is valid if the adiabatic exponent gamma of the explosion gases is between 1.2 and 1.3 or so. Then, t is essentially constant and independent of the amplitude of the pulsation, as seen in Figure 2 of reference (b). The gamma of most of the common explosives is within this range. For other values of gamma, the above assumption is only approximately valid.

The maximum bubble radius in the n-th cycle is, reference (b):

$$(3) \qquad A_{Mn} = 1.733 \ a_M \left(\frac{r_n QW}{Z_n}\right)^{1/3}$$

A surface correction is not necessary for the maximum radius, reference (a) Chapter 8. Assuming that a_M is the same for all cycles, we can eliminate A_{Mn} in (2) by means of

(4) $$\frac{A_{Mn}}{D_n} = \frac{A_{M1}}{D_1}\left(\frac{r_n}{r_1}\right)^{1/3}\left(\frac{Z_1}{Z_n}\right)^{1/3}\frac{D_1}{D_n}.$$

The assumption of an equal a_M is not as good an approximation as that of equal t. Since the significance of the maximum radii in the later pulses is less critical than that of the periods, the accuracy is acceptable for our analysis.

Re-arrangement of (2) yields the following quadratic for the cube root of the bubble energy ratio $(r_n/r_1)^{1/3}$:

(5)

$$\left(\frac{r_n}{r_1}\right)^{2/3}\left[\alpha\frac{A_{M1}}{D_1}\left(\frac{Z_1}{Z_n}\right)^{1/3}\frac{D_1}{D_n}\right]-\left(\frac{r_n}{r_1}\right)^{1/3}+\frac{T_n}{T_1}\left(\frac{Z_n}{Z_1}\right)^{5/6}\left(1-\alpha\frac{A_{M1}}{D_1}\right)=0.$$

In order to find expressions for the ratios of the hydrostatic head Z_n/Z_1 and the depth D_n/D_1, the migration of the bubble must be known. We refer D_n to the center of the n-th bubble maximum and we set for the rise between two such points

(6) $$\frac{D_n - D_{n+1}}{Z_n} = \frac{\Delta Z_n}{Z_n} = \frac{C^*W^{1/2}}{Z_n^{\;2}}\left(1-\alpha\frac{A_{Mn}Z_n}{D_n^{\;2}}\right)\left(\frac{r_n}{r_1}\right)^{1/2}.$$

This formula is commonly used for the migration between the point of explosion and the first minimum. It represents an approximation of Taylor's formula derived in reference (c).

If the assumption is made that the bubble remains spherical, theoretical calculations yield for the coefficient C* the value 132. (For the rise between two successive maxima, C* in (6) would be about twice as large.) Comparison of the theoretical value with that obtained from measured bubble migrations shows the former about 40% too high. Figure 1 illustrates this discrepancy and also illustrates the great scatter of the data.

The magnitude $W^{1/2}/Z^2$ in (6) is related to the Froude Number F which for our purposes is best defined by

$$(7) \qquad F = \frac{A_M}{T^2 g} = \frac{a_M}{0.374 g t^2} \frac{Z^{4/3}}{W^{1/3}} \simeq \left(\frac{Z^4}{W}\right)^{1/3} .$$

Thus

$$(8) \qquad \frac{\Delta Z}{Z} \simeq F^{-3/2} .$$

For the use in conjunction with the quadratic (5), (6) can be brought into the form

$$(9) \qquad \frac{D_n - D_{n+1}}{Z_1} = C\left(\frac{r_n}{r_1}\right)^{1/2} \left(\frac{A_{Mi}}{Z_1}\right)^{3/2} \frac{Z_1}{Z_n} \left[1 - \alpha \frac{A_{M1}}{D_n^{\,2}} Z_n \left(\frac{r_n}{r_1} \frac{Z_1}{Z_n}\right)^{1/3}\right].$$

Then

$$(10) \qquad \frac{Z_n}{Z_1} = 1 - \sum_{j=1}^{n-1} \frac{D_j - D_{j+1}}{Z_1}$$

and

$$(11) \qquad \frac{D_n}{D_1} = 1 - \sum_{j=1}^{n-1} \frac{D_j - D_{j+1}}{Z_1} \frac{Z_1}{D_1}$$

The computation of the reduced migration in the n-th cycle requires only the knowledge of A_{M1}/Z_1 and D_1/Z_1. This provides all relations necessary for the evaluation of (5). The energy ratios r_n/r_1 can be computed if the following information is given:

(a) numerical values for C and α

(b) $\frac{T_n}{T_1}$ as a function of $\frac{A_{M1}}{Z_1}$ and $\frac{D_1}{Z_1}$.

The next paragraphs deal with these data.

III. THE SURFACE CORRECTION TERM

The surface correction term in (1) is given in the literature in various forms. Reference (d) lists the same form as (1), but footnote 12, page 342, of reference (a) shows the surface correction as

$$(12) \qquad 1 - \frac{0.341}{D} \left(\frac{rQW}{Z}\right)^{1/3} F'(x).$$

(Reference (a) erroneously shows a 5/6 power for Z.) For the case of a free water surface and infinitely deep water, $F'(x)$ is unity. With the introduction of (3) and a_M 0.92 (which is appropriate for our conditions), one obtains $\alpha = 0.215$.

Essentially the same value is obtained for the coefficient occurring in the surface correction term for the migration (6) for which we have used the same symbol α. From equation (8.67) of reference (a), this magnitude is found to be 0.2. (There are two misprints in reference (a) at this place: In equation (8.67) the factor 0.2 is omitted. In the preceding equation, the factor should read 0.4 and not 0.2). Reference (d) quotes $\alpha = 1/5$ for the surface correction of the migration.

These values as well as the form of the surface correction term are first order approximations. Figure 2 shows the period constants versus depth of explosion corrected with the use of various coefficients α. These data are from a test series which contributed the most important values to our evaluation. It is seen that $\alpha = 0.1$ makes the period constant independent of depth, i.e. eliminates the surface effect, whereas $\alpha = 0.2$ "overcorrects". Actually, α is not a constant, but depends on A_M/D as well as A_M/Z. Figure 8.21 of reference (a) shows fair agreement for 300 lb TNT charges and Figure 8.20 good agreement for 0.66 lb tetryl charges, if the periods are corrected with $\alpha = 0.2$. But, a closer inspection again reveals an overcorrection, if shallow firing conditions are excluded. It seems that $\alpha = 0.1$ is appropriate for such cases where the bubble is not too close to the surface, as in Figure 2, where $A_M/D < 0.5$. For larger values of A_M/D, i.e. for shallower explosions, a value of α larger than 0.1 is needed for complete correction. This shows that the simple form of the surface correction term in (1) is not sufficient for a precise description of this effect. For the purpose of this paper the complications of a more elaborate relationship would not be worthwhile. In the majority of cases which are of interest here, the bubble is so far away from the water surface, that the value $\alpha = .1$ is applicable. For the instances discussed in paragraph IV, where the bubble break-through is considered, it was found that α had little effect on the location of the bubble maxima and on the times of the break-through. Thus, it seems safe to use $\alpha = 0.1$ for the purposes of this paper.

IV. THE MIGRATION COEFFICIENT

In Figure 1 experimental data on the bubble migration up to the first bubble minimum are compiled. The reduced migration $\Delta Z/Z$ is plotted versus $100\ W^{1/2}/Z^2$. The data stem from various experimental series with a variety of charge weights and explosives. Most of the experimental points were taken from a compilation in reference (f). Only typical points are shown in Figure 1 in order to avoid an overloading of the graph. All values are reduced to an explosive having the properties of TNT with the use of the appropriate equivalent factors. Furthermore, all data are corrected to free field conditions, i.e. an infinite medium. The correction was made as described in Paragraph III of this paper using the factor $\alpha = 0.3$. The results of the 290 lb TNT tests are not very certain, because they were carried out in water of limited depth. The method used to correct for the effect of the bottom consisted of including the term $-\ \alpha\ A_M\ Z/H^2$ into the parentheses of (6), where H is the distance between bottom and point of explosion. This correction is probably not very accurate, but the only one known today.

The migration was measured by the sound ranging method. The point from which the bubble pulse is emitted was located by means of triangulation from a vertical string carrying several pressure gages. One of the difficulties of this method arises from the fact that the pulse is not emitted from a mathematical point, but from the bubble surface which at the moment of the minimum still has a considerable size. The great scatter of data shown in this figure illustrates the difficulties of this measurement and, therefore, the rather approximate nature of the information.

Figure 1 also shows the result of numerical calculations using Taylor's migration formula, reference (c). The points shown have been obtained by rather laborious computations made during World War II by various British agencies quoted in reference (f). Reference (d) stated that on this basis the rise is proportional to $W^{11/24}/Z^{11/6}$. Kennard, reference (e), has noticed that the line drawn through these theoretical points can be equally well represented by a simple relationship which in our notation corresponds to

$$(13) \qquad \left(\frac{\Delta Z}{Z}\right)_{th} = 132\ \frac{W^{1/2}}{Z^2} .$$

The experimentally observed migration can then be represented by

$$(14) \qquad \frac{\Delta Z}{Z} = C^*\ \frac{W^{1/2}}{Z^2} ,$$

where C* is between 80 and 90 depending on the weight one desires to give the migration at shallow or greater depths of explosion respectively. Actually Figure 1 suggests a slightly different functional dependency of W and Z from that of the above equation. It is dubious whether or not the experimental evidence is sufficiently accurate to establish such a functional relationship. In view of the scatter and the difficulties of measuring migration, the form (14) appears to be adequate. Also, it seems that either of the above quoted values of the C* fits the data equally well and can be used with equal confidence.

For the purpose of the present analysis the migration between successive bubble maxima is needed and not that between the point of explosion and the bubble minimum which is shown in Figure 1. The theory of reference (g) (which deals with spherical bubbles) gives the following picture about these migration terms:

Migration between

(a) point of explosion and first maximum

$$(15) \qquad D_o - D_1 = \frac{g\, T_1^2}{3} \left[\ln 2 - 1/2 \right],$$

(b) point of explosion and first minimum

$$(16) \qquad D_o - D^*_1 = \frac{g\, T_1^2}{3} \ln 4\, c_1,$$

(c) first minimum and second maximum

$$(17) \qquad D^*_1 - D_2 = \frac{g\, T_2^2}{3} \left[\ln 2 - 1/2 + \ln 4\, c_1 \right]$$

(d) first maximum and second maximum

$$(18) \qquad D_1 - D_2 = \frac{g}{3} \left[T_1^2 \ln 4\, c_1 + T_2^2 \ln 4\, c_2 + (T_2^2 - T_1^2)(\ln 2 - 1/2) \right]$$

The magnitude c depends on the ratio of maximum to minimum bubble radius:

$$(19) \qquad c = (A_M/A_m)^3 - 1.$$

As before, the subscripts 1 and 2 refer to the first and second cycle of the bubble pulsation respectively.

On the basis of this theory, the migration between the two maxima is just twice the migration between the point of explosion and the first minimum, if the period and the coefficient c are equal in both cycles. Although this result gives a valuable hint, the approximations made in (10), (11), and (6) are apparent: (a) Migration between the point of explosion and the first bubble maximum is neglected in (10) and (11) for simplicity. Relation (15) which is probably rather accurate, since all bubbles remain spherical during the first expansion, indicates that this migration term can indeed be neglected. It is small in comparison with the other terms if the firing conditions are such that several cycles of bubble pulsation occur. It is a poor approximation for large charges exploded so shallow that the breakthrough at the water surface occurs after the first minimum. But, such conditions are not the subject of our study. (b) Since neither T nor c are consistently equal for successive cycles, the factor 2 is not necessarily applicable in (6). But, neither is any constant factor, since the migration between two maxima depends on the parameters of both cycles. It is the simplicity of the analysis as well as the present lack of any better information which justifies approximation (6). In view of these discussions, it appears to be desirable to obtain an independent check for the migration formula (6) and the factor c.*

Such an additional evidence of the bubble migration can be obtained using the analysis developed in this paper and considering the time at which the migrating bubble breaks through the water surface. This event can be conveniently recorded by means of photography of the surface phenomena. A pertinent result is listed in Table I.

TABLE I

Charge Weight: 1590 lb TNT Equivalent

Firing Depth	130	140	150 ft
Time of Bubble Break-through Observed Kinematographically	2.50	3.24	3.25 sec
Time of Latest Bubble Pulse Observed	2.55 3rd pulse	3.32 4th pulse	2.96 sec 4th pulse

Time counts from moment of detonation.

The slight discrepancies between the observed bubble pulses and the times of the bubble breakthrough are probably experimental errors. One should expect the time of breakthrough to be somewhat larger than the time of the pulse. For our purpose the magnitude of the time intervals observed is not important, but only the correlation of these events.

Figures 3a to 3c show the position and size of the bubble maxima calculated with the coefficients C = 3.7, 3.5, and 3.2 for the conditions of Table I. Explosion bubbles are spherical up to the first maximum only. The circles referring to the subsequent cycles in Figures 3a to c must be understood as idealized measures of size. Such bubble shapes are not well defined and can be considered spherical in a crude way only. The numbers shown near the bubble refer to the cycle of the pulsation.

At 130 ft firing depth, the center of the fourth bubble maximum occurs according to these calculations either above or so close to the water surface that a fourth bubble pulse could not have been emitted. This is in agreement with the experimental evidence which shows that the breakthrough must have occurred immediately after the 3rd bubble minimum. It must be visualized that the bubble center jumps at this moment rapidly from the position "3" to the position "4". Figure 3a shows that for C = 3.7 and 3.5 a considerable part of the fourth bubble maximum is above the water surface. This would result in the observed surface disturbances. On the other hand the bubble is too deep at the third cycle for C = 3.2 in order to produce such surface effects. This shows that C must be larger than 3.2.

At 140 ft firing depth, a fourth bubble pulse was observed. For C = 3.7 the bubble is too shallow in the fourth cycle in order to produce a pulse. It is generally assumed that once the depth of the bubble center is less than 90% of the maximum radius no bubble pulse is emitted. (This depth is called the venting depth. We prefer the term "blow-in".) Although this evidence is established for the first bubble maximum only, it is, at least approximately, applicable to the later cycles also. It turns out that in this case C = 3.5 is the largest value for which a fourth bubble pulse can be expected. We have here a rather sensitive criterion for the migration coefficient C.

Figure 3c shows the case of 150 ft firing depth. The calculations using any of the three values for C are compatible with the experimental evidence. One may argue that C = 3.2 again appears low in view of the evidence that the breakthrough essentially coincided with the fourth bubble minimum.

It is obvious that the data available are not sufficient for an unambiguous determination of the migration parameter. However, it is significant that the value C = 3.5 which is the largest value consistent with the evidence of 130 ft firing depth and which holds for the third cycle is compatible with the acoustically measured migrations shown in Figure 1 which hold for the first cycle. C* and C are interrelated by

$$(13) \qquad C^* = J^{3/2} C.$$

On this basis C* = 80 corresponds to C = 1.785. This refers to migrations up to the minimum. The value needed here would be about twice as large, hence 3.57.

For the practical calculations it was decided to use the rounded value

$$C = 3.5.$$

It is realized that this value cannot claim a high degree of accuracy. It is consistent with the experimental evidence available today, but because of the great scatter of the data a considerable uncertainty remains.

In the following analysis, the value of C is somewhat less critical than it might be expected. The bubble energies in the various cycles of the pulsation are, of course, dependent on C. But, when these bubble energies are later used to calculate bubble parameters, some of the uncertainties connected with C will be eliminated. For instance, the periods calculated by this method will almost exactly reflect the input periods. The migration of the bubble and the position of the bubble center of the various bubble maxima are more sensitive to the migration parameter. But, these represent the best information available today.

A sensitive method to check the validity of our method is the observation of the migrating bubble in a high gravity tank. A preliminary study resulted in excellent agreement. A comprehensive test series aimed at a thorough check of the migration parameter is planned.

V. EXPERIMENTAL INPUT

In addition to the surface correction factor α and the bubble migration coefficient C, information is needed on the period ratios T_n/T_1. The form of equation (5) does not require the knowledge of the periods by themselves. Also, the specific firing conditions are not needed for the evaluation of this equation. It is sufficient to know the ratio A_{M1}/D_1 as well as the ratio Z_1/D_1. The nature of this input makes it possible to utilize test results from explosive charges having different explosive material, different charge weights, and different firing conditions. The bulk of the information stems from a test series listed in reference (h). (Denoted as M-series in Figures 4 and 5.) Other data (C-series) are from the files of the E Department of the Naval Ordnance Laboratory.

Figures 4 and 5 show the period ratios T_2/T_1 and T_3/T_1 plotted versus A_{M1}/Z_1. The corresponding values of Z_1/D_1 are given in Figure 4 as curves. The limiting values for great depth ($A_{M1}/Z_1 \rightarrow 0$) are $T_2/T_1 = 0.70$ and $T_3/T_1 = 0.565$.

Information on the fourth bubble period is sparse and uncertain. The curve in Figure 6 is based on the following three values:

TABLE II

A_{M1}/Z_1	D_1/Z_1	T_4/T_1
~ 0	1.0	0.51
0.15	0.81	1.17
0.166	0.82	1.34

The period ratios of non-migrating bubbles ($A_{M1}/Z_1 \sim 0$) show a slight variation for different explosives, reference (j). The values chosen refer to explosives which most closely resemble those employed in the M- and C-series: minol for the second and torpex for the third cycle. The ratio for the fourth cycle is estimated from the TNT result, reference (i), on the basis of the trend exhibited by the different explosives in the preceding cycles.

A crude estimate of r_5/r_4 can be based on the period ratio of non-migrating TNT-bubbles, $T_5/T_4 = 0.927$, reference (i). In absence of any better information the constant value $r_5/r_4 = 0.80$ might be used as a rough approximation.

VI. THE ENERGY OF MIGRATING BUBBLES

The result of the calculations outlined above is shown in Figure 6, where the energy fractions r_n/r_{n-1} are plotted versus the reduced migration. The energy r_n QW refers to the bubble energy at the instant of the n-th bubble maximum. It is well known that the major portion of this energy is potential energy stored in the water during the bubble expansion. The remainder is the internal energy of the gas. Kinetic energy and the energy of migration are negligibly small at this moment.

At each bubble minimum a reduction of the bubble energy takes place. The energy which is lost at this point essentially comprises two terms: (a) energy acoustically radiated by the bubble pulse and (b) dissipated energy. Both of these energy terms depend on the degree of bubble migration. Strong migration reduces the amplitude of the bubble pulse, thus reduces the energy loss due to the acoustic radiation.

For non-migrating bubbles the energy dissipation at the bubble minimum is probably a consequence of the Taylor instability of the bubble interface. With increasing migration and with the consequent decrease of the excess pressure in the bubble near its minimum, the instability decreases. Thus, the intensity of the spray projected into the bubble interior is also decreased and so is the energy dissipation. However, migration causes the inversion of the bubble and an impinging of the upper and lower bubble interfaces. The water hammer and the spray formation connected with it are probably the alternative mechanisms of energy dissipation. These take over with increasing intensity as the effects of the Taylor instability decrease.

These considerations illustrate the important role of migration in the partition of these energy terms. In fact, our analysis clearly shows that the bubble energy fraction depends on the strength of the migration. It also suggests the term which is used in Figure 6 as abscissa.

Figure 6 shows the energy balance for the second cycle at the left hand side. The bubble energy ratio r_2/r_1 is plotted as obtained from our analysis and the input discussed in Paragraph V. This ratio refers to the energy of the bubble at the second maximum. Since this energy is supplied from the bubble energy of the first cycle, the term $1 - r_2/r_1$ represents the energy decrease which occurs at the first bubble minimum. The energy of the pulse which is emitted at the end of the first and the beginning of the second cycle is shown as a dashed curve. (This curve is a rather crude estimate made for the sake of illustration.) The two curves divide the plot into three bands the heights of which represent (a) the bubble energy, (b) dissipated energy, and

(c) the pulse energy. In the dimensionless diagram shown, these three energies add up to unity which is to say that the bubble energy of the first cycle splits into these three terms.

An interesting result is that the bubble energy of the second cycle initially increases with increasing migration, if the other parameters of the explosion are held constant. In view of the preceding comments, this is to be expected. It is also seen that the dissipated energy is roughly constant over a considerable range of migration intensity. This is surprising, since two different mechanisms of energy dissipation are probably involved, both of which depend on migration. Although one of them decreases and the other increases with increasing migration, a roughly constant sum of these factors, as suggested by Figure 6, was not necessarily to be expected.

For very strong migration, the bubble inversion and the consequent energy dissipation appear to be so violent that a decrease of the bubble energy in the second cycle results. For such conditions the energy radiated by the bubble pulse is practically negligible.

The most important outcome of these calculations is that the energy ratios given in Figure 6 permit the calculation of the parameters of migrating bubbles for almost all explosion conditions of practical interest. (Nuclear explosions are, of course, not included.) The calculations can be extended with reasonable confidence up to the end of the fourth cycle. Estimates for the fifth cycle are possible with the use of the approximate energy ratio $r_5/r_4 = 0.8$.

The conditions at the bubble minimum, its size and shape as well as its energy of translation are not covered in this analysis. However, it is possible to calculate the position of the bubble minimum simply by assuming it halfway between successive bubble maxima.

According to the present state of knowledge the energy curves are applicable to most explosives. Some deviations may be expected for deep explosions, where only slight migration takes place. The evidence of non-migrating bubbles, reference (j), shows a slight trend in the period ratios for explosives of different compositions. But, the results of the M- and C-series give no indication of such a trend. If desired the effect of different explosives can be accounted for in Figure 6 by constructing a new curve which passes through the appropriate value at $A_M/Z = 0$ and subsequently merges into the old curve. But, in most cases, such a precaution will not be necessary.

The practical application of the information obtained here, is discussed in NOLTR 62-184. In this paper graphs are presented which permit a convenient reading of the bubble parameters for a wide range of conditions.

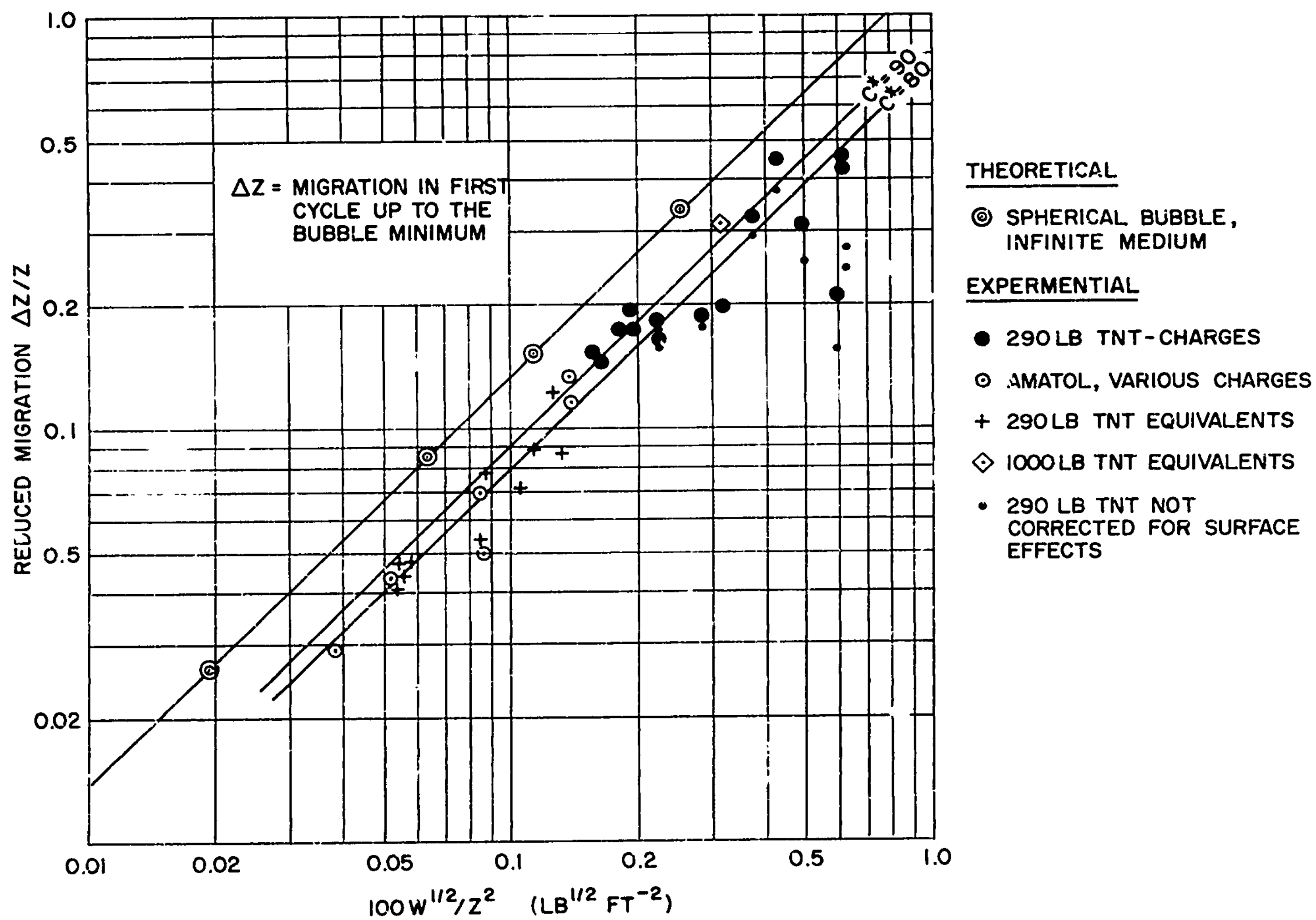

FIG.1 EVIDENCE ON BUBBLE MIGRATION

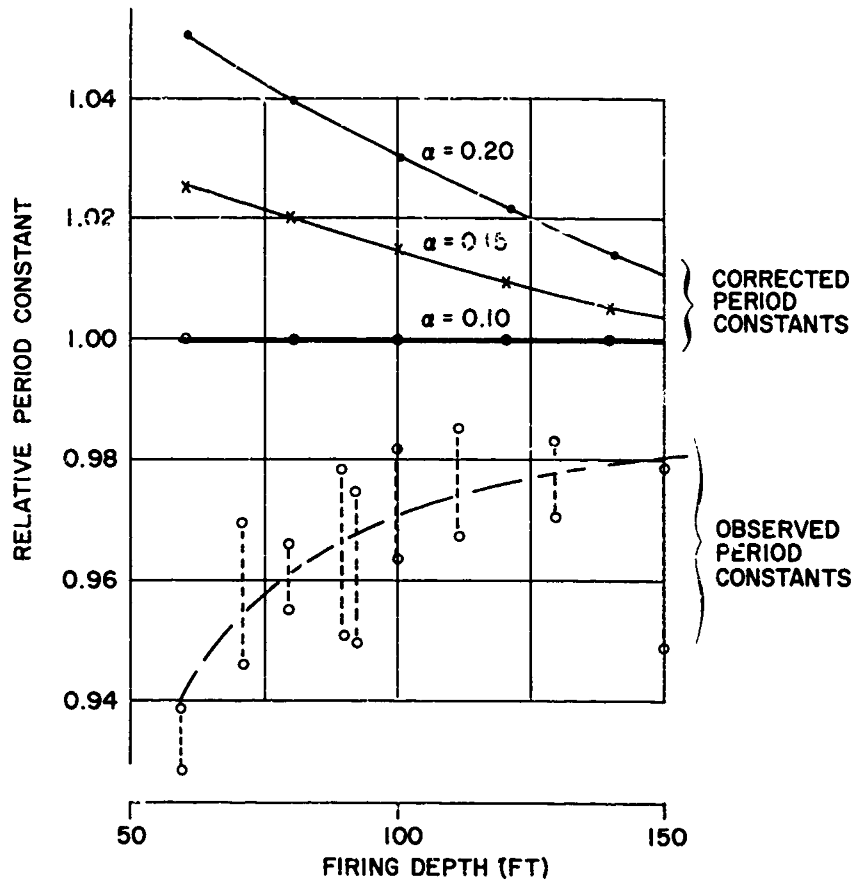

FIG. 2 SURFACE CORRECTION OF THE FIRST BUBBLE PERIOD. DATA ARE FROM CHARGES OF 1580 LB TNT EQUIVALENT. THE DASHED LINE BETWEEN THE POINTS INDICATES THE UNCERTAINTY IN THE READING OF THE PERIOD.

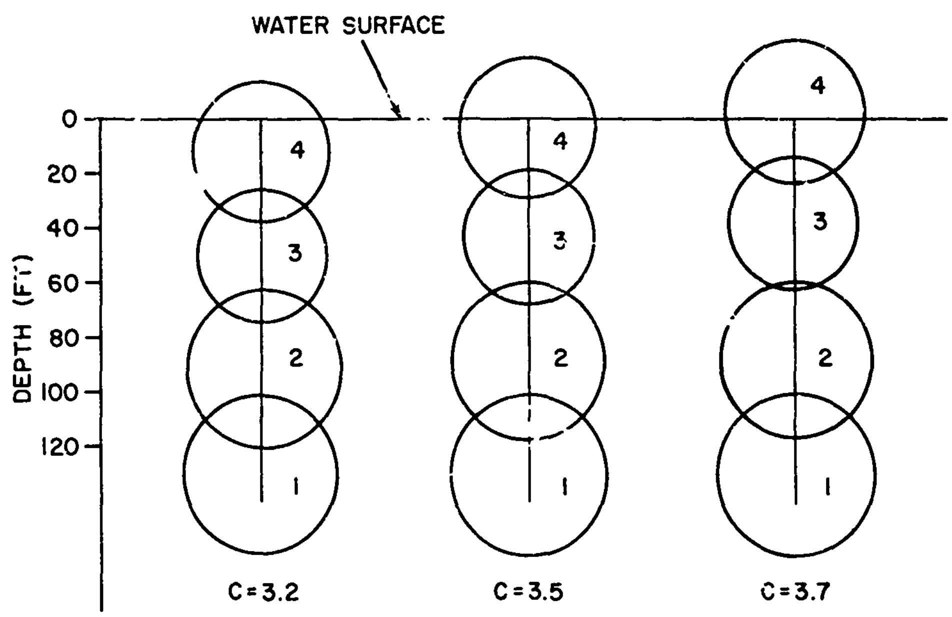

FIG. 3a POSITIONS AND RADII OF THE BUBBLE MAXIMA CALCULATED FOR THREE VALUES OF THE MIGRATION PARAMETER C. CHARGE WEIGHT 1580 LB-TNT EQUIVALENT, FIRING DEPTH 130 FEET.

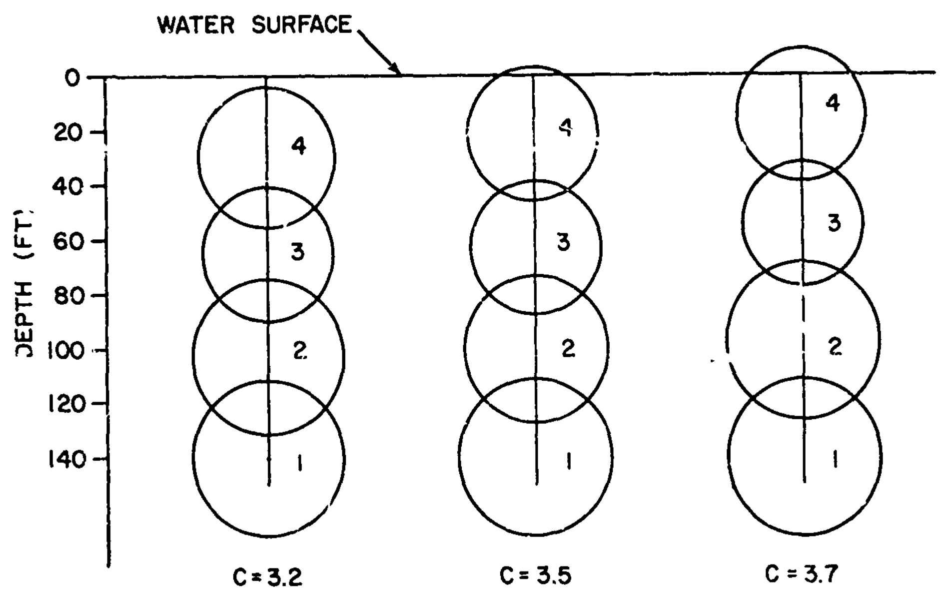

FIG. 3b POSITIONS AND RADII OF THE BUBBLE MAXIMA CALCULATED FOR THREE VALUES OF THE MIGRATION PARAMETER C. CHARGE WEIGHT 1580 LB -TNT EQUIVALENT, FIRING DEPTH 140 FEET.

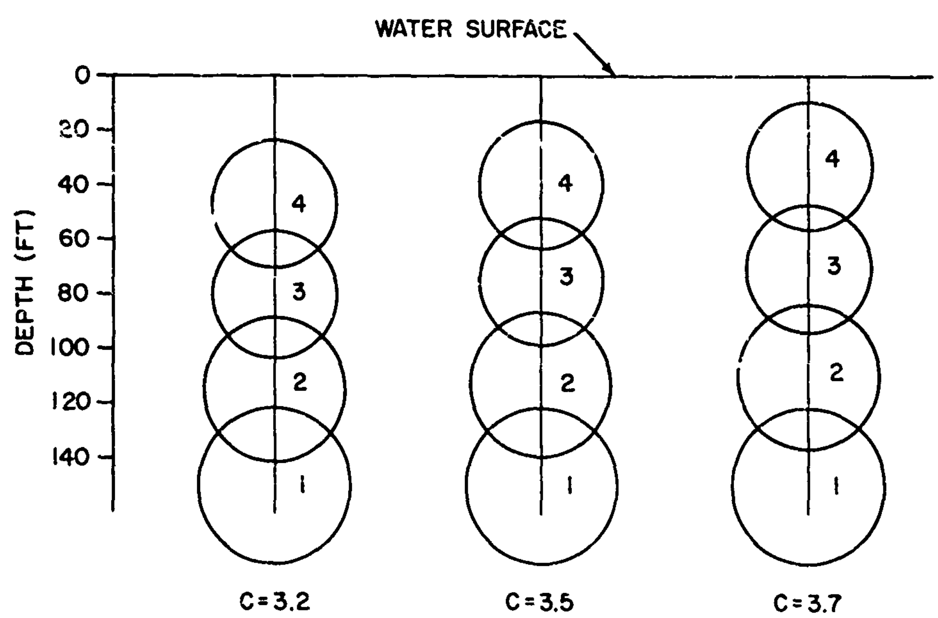

FIG. 3c POSITIONS AND RADII OF THE BUBBLE MAXIMA CALCULATED FOR THREE VALUES OF THE MIGRATION PARAMETER C. CHARGE WEIGHT 1580 LB—TNT EQUIVALENT, FIRING DEPTH 150 FEET.

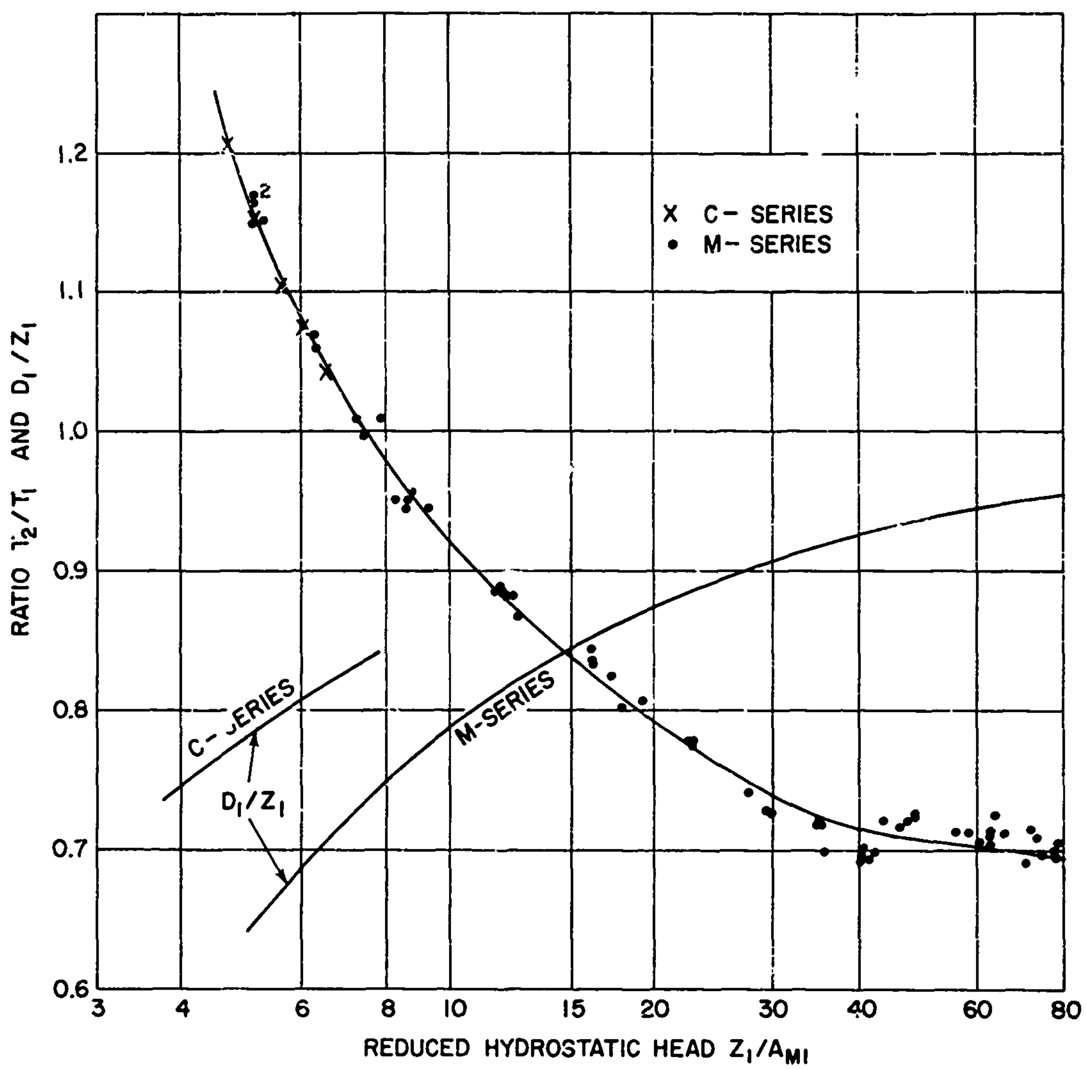

FIG 4 PERIOD RATIO T_2/T_1 AND RATIO OF FIRING DEPTH TO HYDROSTATIC HEAD. ABSCISSA IS THE HYDROSTATIC HEAD REDUCED BY THE FIRST MAXIMUM BUBBLE RADIUS.

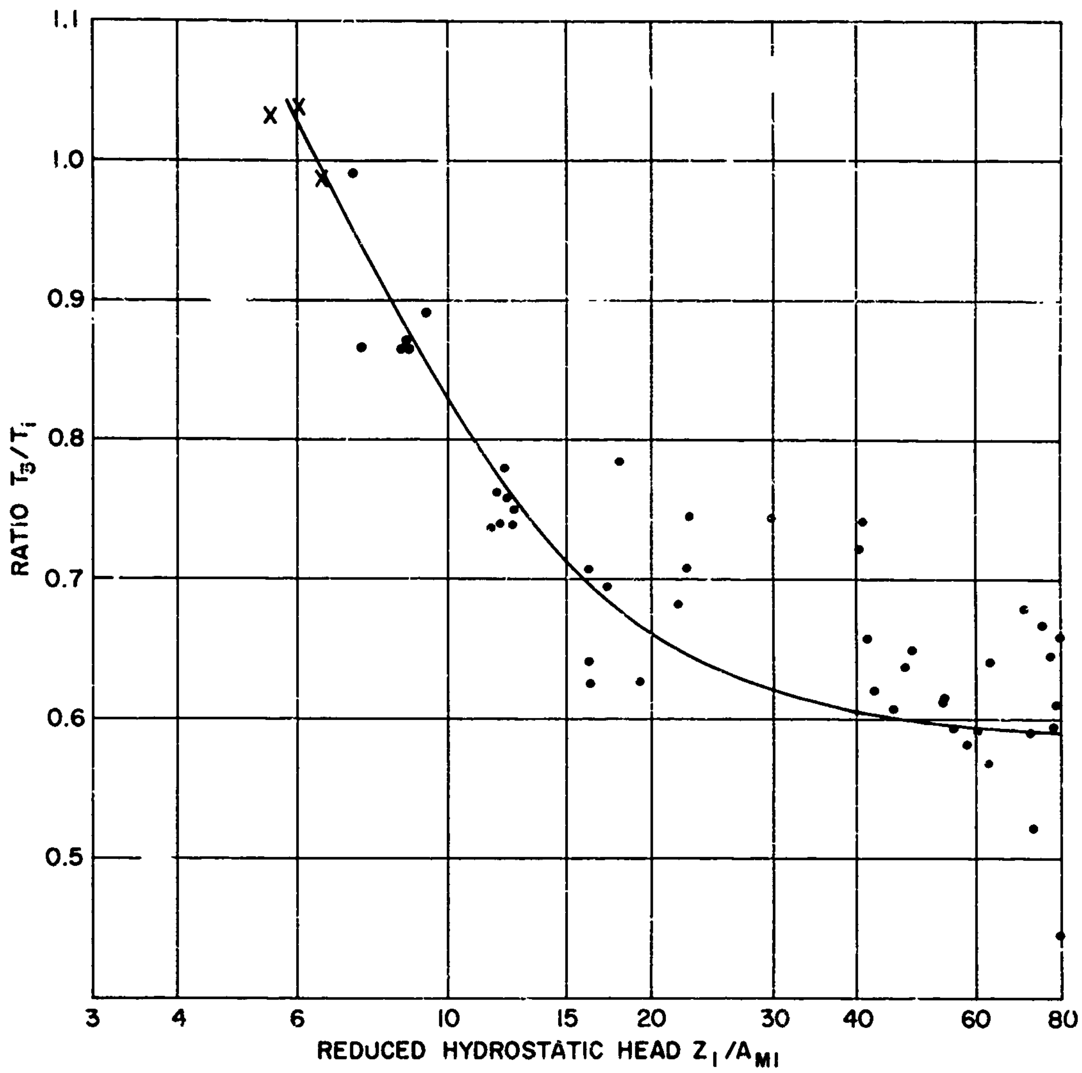

FIG. 5 PERIOD RATIO T_3/T_1. THE MAGNITUDES USED AS ABSCISSA REFER TO THE FIRST CYCLE OF PULSATION

LIST OF SYMBOLS

A_M	=	Maximum Bubble Radius (ft)
A_m	=	Minimum Bubble Radius (ft)
a_M	=	Reduced Maximum Bubble Radius (dimensionless)
c	=	See equation (19)
C*	=	Migration Coefficient ($ft^2/lb^{1/2}$)
C		Migration Coefficient (dimensionless)
D	=	Depth of Bubble Center at Bubble Maximum (ft)
D_o	=	Firing Depth (ft)
F	=	Froude Number (Dimensionless)
F'(x)	=	Surface Correction Function (dimensionless)
g	=	Acceleration of Gravity (ft/sec^2)
H	=	Distance to Bottom (ft)
J	=	Radius Constant ($ft^{4/3}/lb^{1/3}$)
K	=	Period Constant (sec $ft^{5/6}/lb^{1/3}$)
n	=	Subscript Referring to Cycle of Pulsation
Q	=	Chemical Energy per Unit Mass of Explosive (cal/lb)
r	=	Energy Fraction Referring to Bubble (dimensionless)
rQW	=	Bubble Energy (cal)
T	=	Period (sec)
t	=	Reduced Period (dimensionless)
W	=	Charge Weight (lb)
x	=	(D-HO/(D+H) (dimensionless)

Z = Total Hydrostatic Head = D + 33 ft for sea water

Z_o = Total Hydrostatic Head at firing depth (ft)

α = Surface Correction Factor (dimensionless)

ϵ = rQ

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Explosions	EXPS	Analysis	ANAL		
Migration	MIGR	Cycles	CYCE		
Bubbles	BUBB	Oscillation	OSCA		
Phenomena	PHEO	Maximum	MAXM		
Parameters	PARA	Radius	RAUS		
Gravity	GRAV	Hydrodynamics	HYDN		
Buoyancy	BUOA				
Energy	ENER				
Pulsation	PULS				
Duration	DURT				
Variation	VART				

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Naval Ordnance Laboratory, White Oak, Md.
(NAVORD report 4185)
UNDERWATER EXPLOSION PHENOMENA: THE PARAMETERS OF MIGRATING BUBBLES (U), by Hans G. Snay. 12 Oct. 1962. 24p. Tasks 301-664/43003/01040 and RUME 3-E-000/212-1/WF008-10-004, P.A. 002. UNCLASSIFIED

The migration of underwater explosion bubbles caused by gravity (buoyancy) affects the energy of the pulsation as well as period and maximum radius in the second and subsequent cycles. Experimental data on periods of explosions in various depths are analyzed, and the bubble energy is determined for five cycles of the oscillation as a function of the strength of migration. The result is given in dimensionless form which permits the calculation of periods, maximum radii, and migration for a wide range of conditions.

1. Explosions, Underwater
2. Bubbles, Explosion
3. Bubbles - Migration
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